A fuzzy dynamic wave routing model

R. Gopakumar\textsuperscript{1} and P. P. Mujumdar\textsuperscript{2}\textsuperscript{*}

\textsuperscript{1} Department of Civil Engineering, Government Engineering College, Thrissur—680009, India
\textsuperscript{2} Department of Civil Engineering, Indian Institute of Science, Bangalore—560012, India

Abstract:
In this paper a fuzzy dynamic wave routing model (FDWRM) for unsteady flow simulation in open channels is presented. The continuity equation of the dynamic wave routing model is preserved in its original form while the momentum equation is replaced by a fuzzy rule based model which is developed on the principle that during unsteady flow the disturbances in the form of discontinuities in the gradient of the physical parameters will propagate along the characteristics with a velocity equal to that of velocity of the shallow water wave. The model gets rid off the assumptions associated with the momentum equation by replacing it with the fuzzy rule based model. It overcomes the necessity of calculating friction slope ($S_f$) in flow routing and hence the associated uncertainties are eliminated. The robustness of the fuzzy rule based model enables the FDWRM to march the solution even in regions where the aforementioned assumptions are violated. Also the model can be used for flow routing in curved channels. When the model is applied to hypothetical flood routing problems in a river it is observed that the results are comparable to those of an implicit numerical model (INM) which solves the dynamic wave equations using an implicit numerical scheme. The model is also applied to a real case of flow routing in a field canal. The results match well with the measured data and the model performs better than the INM. Copyright © 2007 John Wiley & Sons, Ltd.

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INTRODUCTION

In open channels the discharge and stage hydrographs at various locations are determined by flow routing. A comprehensive review of various methods of flow routing is given by Singh (2004). The most accurate method of flow routing is the dynamic wave routing (Chow \textit{et al.}, 1988) in which the Saint Venant equations are solved for specified initial and boundary conditions. For a wide rectangular channel conveying water these equations can be written as follows (Liggett, 1975):

(1) Continuity equation.
\[ \frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = 0 \] (1)

(2) Momentum equation.
\[ \frac{\partial q}{\partial t} + \frac{\partial}{\partial x} \left[ q^2 + \frac{gh^2}{2} \right] - ghS_0 + ghS_f = 0 \] (2)

where $q$ is discharge per unit width, $h$ is flow depth, $S_0$ is bed slope, $S_f$ is friction slope, $g$ is acceleration due to gravity, $t$ is time variable and $x$ is space variable.

Conventionally Equations (1) and (2) are solved using implicit numerical models (INMs) to obtain values of $h$ and $q$ at specified locations along the channel. A major limitation of such numerical models is that due to uncertainties involved in the estimation of the friction slope ($S_f$) in the momentum equation. The friction slope is usually computed with Manning’s equation wherein roughness coefficient ($n$) is estimated based on several channel characteristics such as surface roughness, size, shape, vegetation, irregularity, alignment, stage, discharge, etc. (Chow, 1959). The roughness coefficient is one of the main variables being used in calibrating the INM (US Army Corps of Engineers, 2002). But accurate estimation of $n$ is very difficult due to its dependence on the several factors stated earlier. Another limitation of the model is that it is sensitive to the various hydraulic processes that violate the underlying assumptions of the momentum equation. For example, a transition from sub-critical to super-critical flow anywhere in the computational domain can cause a program failure. ASCE Task Committee (1993) describes various circumstances under which the program failure can occur. A third limitation is that the momentum equation is strictly valid only for straight reaches of the channel whereas most field channels are not straight. These restrictions are not applied to the continuity equation. Hence a model based on the continuity equation alone; with an appropriate data-driven fuzzy rule based substitution for the momentum equation will be capable of solving the earlier problems. Such an approach has been used earlier by Bardossy and Disse (1993) in modeling infiltration.

Fuzzy rule based models are suitable when information about the physical process is vague and data available is

\textsuperscript{*} Correspondence to: P. P. Mujumdar, Department of Civil Engineering, Indian Institute of Science, Bangalore—560012, India. E-mail: pradep@civil.iisc.ernet.in

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scarce. The physical information is utilized in development of the rules which are then tuned using the available data. Details of fuzzy numbers, membership functions and fuzzy rules can be found in the literature (Bardossy and Disse, 1993; Dou et al., 1999; Ross, 1995). A good introduction to practical aspects of fuzzy rule based modelling has been given by Russel and Campbell (1996) and Kisi et al. (2006). They highlighted the robustness of the fuzzy rule based models. Other advantages of these models include their transparent structure and simplicity in development.

After the concept of fuzzy logic being first introduced by Zadeh (1965) a lot of research has been done towards fuzzy logic-based modelling of various hydrological processes (Fujita et al., 1992; Bardossy and Disse, 1993; Schulz and Huwe, 1997; Dou et al., 1999; Huncha et al., 2001; Özelkan and Duckstein, 2001; Xiong et al., 2001; Mahabir et al., 2003; Samanta and Mackay, 2003; Nayak et al., 2005; Inamdar et al., 2006; Sen and Altunkaynak, 2006). But it has been found that very little work has been done in flood routing. In this paper a fuzzy dynamic wave routing model (FDWRM) is presented in which the momentum equation of the dynamic wave routing model is replaced by a fuzzy rule based model while retaining the continuity equation in its complete form.

**MODEL DEVELOPMENT**

The continuity equation (Equation (1)) is discretized at node \((i, j)\) in the \(x - t\) domain, using the Lax diffusive scheme (Chaudhry, 1993), which is an explicit finite difference scheme, as follows:

\[
\frac{1}{\Delta h} \left[ q_{i,j}^{t+1} - 0.5 \times (q_{i,j}^{t} + q_{i,j}^{t+1}) \right] + \frac{1}{2\Delta x} \left[ \Delta q_{i,j}^{t+1} - \Delta q_{i,j}^{t} \right] = 0
\]

(3)

The momentum equation (Equation (2)) is replaced by fuzzy rule based model. Here it is assumed that no flow reversal takes place in the system which is usually true for the cases of flow in irrigation canals and floods in natural rivers. During unsteady flow the disturbances in the form of discontinuities in the gradient of the physical parameters will propagate along the characteristics with a velocity equal to that of velocity of the shallow water wave \((V_w)\) (Cunge et al., 1980). These discontinuities in the gradient of the parameters are computed as the difference between their final gradient and initial steady flow gradient wherein the respective gradients are obtained in terms of the difference in the parameter values between adjacent sections. Thus the discontinuity in the gradient of \(h\) is obtained as \((\Delta h/\Delta x)\) and that of \(q\) is obtained as \((\Delta q/\Delta x)\) where \(\Delta h\) and \(\Delta q\) are differences in the increments of \(h\) and \(q\) respectively between present time level and initial steady flow level. The ratio \((\Delta q/\Delta h)\) will give a measure of wave velocity \((V_w)\) in the wide rectangular channel under consideration (Chow, 1959). If it is assumed that \(V_w\) is constant in the range of flow depths being considered then a set of crisp rules can be written between the sections relating \(\Delta h\) and \(\Delta q\) as IF \(\Delta h\) is low THEN \(\Delta q\) is low, IF \(\Delta h\) is medium THEN \(\Delta q\) is medium, IF \(\Delta h\) is high THEN \(\Delta q\) is high, etc. where low, medium, high etc. are quantitative variables. The variation of \(V_w\) with respect to flow depth is accounted for by considering \(\Delta h\) and \(\Delta q\) as fuzzy sets. Then the aforementioned rules become fuzzy and low, medium, high, etc. become linguistic variables. This is the principle underlying development of the fuzzy rule based model. The rules become fuzzier with increase in backwater effects.

The antecedent \(\Delta h\) and the consequent \(\Delta q\) are assumed to be triangular fuzzy numbers. As accuracy of fuzzy rule based models has been reported to be less sensitive to shape of their membership functions (Sugeno and Yasukawa, 1993) this assumption is considered reasonable. The training set required for development of the rules may be obtained either from physical unsteady flow measurement of \(h\) and \(q\) at specified locations along the channel for various inflow discharges or from output of an implicit numerical model (INM) which solve Equations (1) and (2) for these discharges. If the entire reach of the channel does not show heterogeneity with respect to bed slope, surface roughness and cross section then only one set of rules will be sufficient for the entire reach. Otherwise different sets of fuzzy rules will be necessary for each of the homogenous sub reaches. Here homogeneity is not a strict requirement because minor variations (which are possible in real life situations) will be taken care of by the fuzzy rule based model. The procedure given by Bardossy and Disse (1993) is used for assessment of fuzzy rules and membership functions from the training sets. The Mamdani implication method of inference (Mamdani and Assilian, 1975) and the Centroid method of defuzzification (Ross, 1995) are adopted to obtain crisp values for the aggregated output.

A major advantage of the earlier fuzzy formulation is that the number of variables involved is only two. Also as \(\Delta h\) and \(\Delta q\), instead of \(h\) and \(q\), are taken as the input and output variables the number of fuzzy sets per variable will be much less. These will help in preventing the rule explosion described by See and Openshaw (1999). As the number of rules is less, the calibration of the model will be easier with the limited available data.

**Initial and boundary conditions**

Values of \(h\) and \(q\) at the beginning of the time step are to be specified at all the nodes along the channel as initial conditions. The three boundary conditions required by the model are the inflow discharge hydrograph at the upstream boundary, the stage hydrograph at the upstream boundary and the stage hydrograph at the downstream boundary.

**Method of solution**

- **Step 1.** From the known initial conditions and using the discretized continuity equation (Equation (3)) the
values of \( h \) at the end of the time step are obtained at all the nodes. As Equation (3) is based on an explicit finite difference scheme the time step should be chosen satisfying the CFL (Courant–Frederich–Levy) condition (Chaudhry, 1987).

- Step 2. Using the \( h \) values obtained in Step 1 along with its initial values and applying the upstream and downstream boundary stage hydrograph ordinates, the \( \Delta h \) values between adjacent nodes are calculated proceeding from upstream to downstream.
- Step 3. Corresponding \( \Delta q \) values between the nodes are obtained by running the fuzzy rule based model.
- Step 4. The values of \( q \) at each node at the end of the time step are obtained by using the upstream boundary discharge hydrograph ordinates along with the initial \( q \) values and the \( \Delta q \) values obtained in Step 3.

Thus values of \( q \) and \( h \) at the end of the time step are obtained at all the nodes. They are used as the initial conditions for the next time step and the Steps (1)–(4) are repeated till the entire time period is covered.

The FDWRM is applicable equally for sub-critical as well as super-critical flows. In sub-critical flow disturbances from the upstream boundary move along the positive characteristics in the downstream direction and will be modified by disturbances originating from the downstream boundary which move along the negative characteristics in the upstream direction. In super-critical flow the disturbances move in the downstream direction only.

**MODEL APPLICATION**

To demonstrate the potentials of the FDWRM, it is applied to simulate hypothetical flood routing problems in a wide rectangular river and also a realistic flow routing problem in a field canal.

**Hypothetical flood routing in a wide rectangular river**

This application of the model is intended to test its capability to replace the implicit numerical model (INM). Flood routing in a 32 km long wide rectangular river with bed slope \( (S_0) \) equal to 0.00059 is considered for study. Uniform flow exists initially with depth 5.0 m and base flow \( (q_b) \) 11.6 m\(^3\) s\(^{-1}\) m\(^{-1}\) width. The fuzzy rules are derived from the training set obtained by solving Equations (1) and (2) using the finite difference INM described by Chow et al. (1988) which make use of the Preissmann scheme (Abbott, 1979) for discretization. Friction slope is predicted using Manning’s equation with roughness coefficient \( n \) equal to 0.031. The upstream discharge hydrograph is given by:

\[
q(t) = q_b + \frac{q_b}{2} \left( 1 - \cos \frac{\pi}{t_p} \right) \quad \text{for } t \leq t_p
\]

\[
q(t) = q_b + \frac{q_b}{2} \left( 1 - \cos \frac{\pi}{t_b - t_p} \right) \quad \text{for } t_p < t \leq t_b
\]

where \( t_b \) is the time base and \( t_p \) is the time to peak taken equal to \( t_b/2 \). The downstream boundary stage-discharge relationship is given by:

\[
q_{end} = \frac{1}{n} \left( h_{end}^{3/3} \right) (S_0^{1/2})
\]

where \( q_{end} \) is the discharge per unit width at the downstream end and \( h_{end} \) is the flow depth at the downstream end. The INM is run for different inflow flood discharges obtained for various values of \( t_b \). From the output \( h \) and \( q \) values, the corresponding \( \Delta h \) and \( \Delta q \) values are computed for each of the adjacent sections. It is found that the \( \Delta h \) ranges from 0.01 m to 0.18 m and the \( \Delta q \) ranges from 0.05 to 0.73 m\(^3\) s\(^{-1}\) m\(^{-1}\) width.

Membership function plots of the input \( \Delta h \) and the output \( \Delta q \) are shown in Figure 1. In Figure 1 ‘L’ implies ‘low’ values and ‘H’ implies ‘high’ values. Higher the

![Figure 1. Fuzzy membership functions for hypothetical flood routing in the river: (a) input membership functions and (b) output membership functions](image-url)
coefficient of ‘L’ and ‘H’ higher will be their gradation towards ‘low’ and ‘high’ values respectively. Parameters of the membership functions have been optimized using the neuro-fuzzy model NEFCON proposed by Nürnberger et al. (1999).

A total of 18 rules are derived as follows:

- Rule 1. IF \( \Delta h \) is 9L THEN \( \Delta q \) is 9L
- Rule 2. IF \( \Delta h \) is 8L THEN \( \Delta q \) is 8L
- Rule 3. IF \( \Delta h \) is 7L THEN \( \Delta q \) is 7L
- Rule 4. IF \( \Delta h \) is 6L THEN \( \Delta q \) is 6L
- Rule 5. IF \( \Delta h \) is 5L THEN \( \Delta q \) is 5L
- Rule 6. IF \( \Delta h \) is 4L THEN \( \Delta q \) is 4L
- Rule 7. IF \( \Delta h \) is 3L THEN \( \Delta q \) is 3L
- Rule 8. IF \( \Delta h \) is 2L THEN \( \Delta q \) is 2L
- Rule 9. IF \( \Delta h \) is L THEN \( \Delta q \) is L
- Rule 10. IF \( \Delta h \) is H THEN \( \Delta q \) is H
- Rule 11. IF \( \Delta h \) is 2H THEN \( \Delta q \) is 2H
- Rule 12. IF \( \Delta h \) is 3H THEN \( \Delta q \) is 3H
- Rule 13. IF \( \Delta h \) is 4H THEN \( \Delta q \) is 4H
- Rule 14. IF \( \Delta h \) is 5H THEN \( \Delta q \) is 5H
- Rule 15. IF \( \Delta h \) is 6H THEN \( \Delta q \) is 6H
- Rule 16. IF \( \Delta h \) is 7H THEN \( \Delta q \) is 7H
- Rule 17. IF \( \Delta h \) is 8H THEN \( \Delta q \) is 8H
- Rule 18. IF \( \Delta h \) is 9H THEN \( \Delta q \) is 9H

The developed model is applied to three cases of flood routing in the river. The cases vary with respect to nature of the back water effects present during simulation. With increase in backwater effects the rules become fuzzier and hence reduction in accuracy of the model results can be expected.

**Case I.** This is a case where very gradual changes in the discharge values are applied at the upstream boundary and the back water effects are negligible. The discharge hydrograph given by Equation (4) with value of \( t_b \) equal to 30 days is applied at the upstream boundary. Flood routing has been done using both the FDWRM and the INM. The initial condition for both the models corresponds to the uniform flow parameters existing initially. For the INM the discharge hydrograph applied at the upstream end and the stage-discharge relationship (Equation (5)) at the downstream end are used as the boundary conditions. The upstream and downstream boundary stage hydrographs used for the FDWRM are shown in Figure 2.

Stage and discharge hydrographs obtained at a section distant 16 km from the upstream end and discharge hydrograph obtained at the downstream end of the channel by applying both the models are shown in Figure 3. From Figure 3 it can be seen that the FDWRM outputs match very well with those given by the INM. As back water effects are negligible these results are as expected. Hence it can be concluded that for this case the FDWRM can very well replace the INM.

**Case 2.** In this case comparatively rapid changes in the discharge values are imposed at the upstream boundary and therefore back water effects are present during simulation. The discharge hydrograph given by:

\[
q(t) = q_b + \frac{q_b}{8} \left(1 - \cos \frac{t}{t_p} \right) \quad \text{for } t \leq t_p
\]

\[
q(t) = q_b + \frac{q_b}{8} \left(1 - \cos \frac{t_b}{t_b - t_p} \right) \quad \text{for } t_p < t \leq t_b
\]

with \( t_b \) equal to 30 h is applied at the upstream boundary. Here also food routing is done using both the FDWRM and the INM with the same uniform flow initial condition as that specified in Case 1. The upstream discharge hydrograph (Equation (6)) and the downstream stage-discharge relationship (Equation (5)) are used as boundary conditions for the INM. Upstream and downstream boundary stage hydrographs used for the FDWRM are shown in Figure 4.

Stage and discharge hydrographs at the 16 km section and discharge hydrograph at the downstream end of the channel, obtained from the models, are shown in Figure 5. It can be seen that the FDWRM outputs match well with those given by the INM but the accuracy is

![Figure 2](image-url) Boundary conditions for flood routing in the river (Case 1): (a) stage hydrograph at upstream boundary and (b) stage hydrograph at downstream boundary

![Figure 3](image-url) Stage and discharge hydrographs obtained at a section distant 16 km from the upstream end and discharge hydrograph obtained at the downstream end of the channel by applying both the models are shown in Figure 3.
Figure 3. Simulation results of flood routing in the river (Case 1): (a) stage hydrograph at 16 km, (b) discharge hydrograph at 16 km, and (c) discharge hydrograph at downstream end

Figure 4. Boundary conditions for flood routing in the river (Case 2): (a) stage hydrograph at upstream boundary and (b) stage hydrograph at downstream boundary

Weir provided at the downstream end. Height of the weir is 3 m with coefficient of discharge ($C_d$) taken equal to 1.0. The initial condition is a steady gradually varied flow profile (backwater curve) with flow depth ranging from 5.48 m at the downstream end to 5.01 m at a section 8 km from the downstream end. The initial steady discharge is $11.6 \, \text{m}^3 \, \text{s}^{-1} \, \text{m}^{-1} \, \text{width} (q_b)$.

For this case also the discharge hydrograph given by Equation (6) with $t_b$ equal to 30 h is applied at the upstream boundary. Both the FDWRM and the INM are used to simulate the flood movement. Same initial condition (the steady gradually varied flow profile) is used for both the models. The discharge hydrograph applied at the upstream end and the free flow rectangular weir formula given by:

$$q_{end} = \frac{2}{3} C_d \sqrt{2g(h_{end}^{3/2})}$$

at the downstream end are used as the boundary conditions for the INM. Upstream and downstream boundary stage hydrographs used for the FDWRM are shown in Figure 6. Maximum back water effect occurs in this case as compared to the previous cases.
Figure 5. Simulation results of flood routing in the river (Case 2): (a) stage hydrograph at 16 km, (b) discharge hydrograph at 16 km, and (c) discharge hydrograph at downstream end

Stage and discharge hydrographs at the 16 km section and discharge hydrograph at the downstream end, computed using the models, are shown in Figure 7. From Figure 7 it can be seen that the accuracy is least in this case as compared to the previous cases. This is due to the severe backwater effects present during the simulation. Even then reasonably good matching can be observed between the two model results which again prove that the FDWRM is a good substitute for the INM.

Flow routing in a field canal

Here the FDWRM is tested for its capability in handling field flow routing problems. For this purpose, historical unsteady flow data in a 30 km long irrigation canal, corresponding to upstream gate operations, have been collected. Bed width of the canal is 50 m throughout the length and is rectangular in cross section. The canal is not straight and it consists of several bends. Its bed slope is 0-0035. Training sets required for development of the fuzzy rules in the FDWRM are derived from the historical data. From the training sets it is found that the input (Δh) ranges from 0-06 m to 0-36 m and the output (Δq) ranges from 0-1 to 0-68 m³ s⁻¹ m⁻¹ width. A total of nine membership functions are derived for the input as well as for the output and nine rules are derived connecting the input and the output. For this case also the parameters of the membership functions have been optimized using the neuro-fuzzy model NEFCON proposed by Nürnberg et al. (1999). Nature of the membership functions and
of the rules derived is similar to those developed for the hypothetical flood routing problems in the river.

The developed FDWRM as well as the INM are applied to a typical flow routing problem in the canal for which the measured unsteady flow data at three sections are given in Table I. The initial condition for both the models corresponds to uniform flow with discharge 0.67 m$^3$ s$^{-1}$ m$^{-1}$ width and flow depth 0.6 m. For the FDWRM the upstream stage and discharge hydrographs and the downstream stage hydrograph are used as the boundary conditions while for the INM the discharge hydrograph at the upstream end and the stage hydrograph at the downstream end are used as the boundary conditions. For the INM it is necessary to assume that the entire reach of the canal is straight. Also, the friction slope required in the INM is computed using Manning’s equation with roughness coefficient $n$ equal to 0.02. Neither of these is required for the FDWRM. The simulation results are given in Figure 8.

Stage hydrograph at 15 km from upstream, obtained using the models along with the measured data, is shown in Figure 8(a). The kinks in the measured data are due to rapid changes in the discharge values applied at the upstream boundary. From Figure 8(a) it can be seen that

<table>
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<th>Table I. Measured unsteady flow data in the irrigation canal</th>
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<td>Time (h)</td>
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CONCLUSIONS

A FDWRM, in which the momentum equation of the traditional dynamic wave routing model is replaced by a fuzzy rule based model while retaining the continuity equation in its complete form, is presented in this paper. By replacing the momentum equation by the robust fuzzy rule based model the associated assumptions are eliminated, thus making the model applicable to any general channel. As there is no need of estimating the friction slope, the model gets rid off the involved uncertainties. For demonstration the model was applied to two situations: hypothetical flood routing problems in a wide rectangular river and a realistic flow routing problem in a field canal. The results obtained were compared to those of an INM, which solves the dynamic wave equations using an implicit finite difference scheme.

Three cases were considered under hypothetical flood routing in the river which varies essentially in the nature of backwater effects present during the simulation. The backwater effects were negligible for the first case, mild for the second case and severe for the third case. The relations between input and output variables become fuzzier with increase in backwater effects and corresponding reduction in accuracy of the model results has been observed. A reasonably good matching of the model results with that of the implicit numerical model has been obtained for all the three cases indicating that the model can effectively replace the INM. For the case of flow routing in the field canal the model results match well with measured data and also are much better than that of the implicit numerical model.

REFERENCES


