Multi-Mechanism Modeling of Fatigue Crack Growth under Complex Load Sequences

R. Sunder, BiSS Research
Bangalore, India, +91 9880-432322
rs@biss.in

IISc, Bangalore (Centenary Lecture, Civil Engg)
Jan 15, 2009
Acknowledgement

1. Experimental facilities at AFRL (WPAFB) & BiSS Research
2. Discussions and worldwide seminar feedback over about 10 years
3. Support and understanding of colleagues

BiSS Research is entirely funded by BiSS (P) Ltd
Sequence

1. Brief introduction to Metal Fatigue
2. Non-linear cumulative damage
3. Significance of interaction between near-tip hydrostatic stress and environment
4. Modeling crack growth under complex load sequences
5. Summary & Conclusion

“The ultimate tragedy of analytical effort: When an absurd fact demolishes a perfectly elegant model” – [Russian science humour]
The S-N Curve – Failure can occur even at small stresses, provided they are cyclic.

“Fatigue Limit” – Infinite life

Increased mean stress

<table>
<thead>
<tr>
<th>Reference number</th>
<th>Greatest tension on fibres of test-bar (Centners per German square inch)</th>
<th>Number of applications of strain before fracture took place</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 2 3 4</td>
<td>480 450 420 400</td>
<td>138,600 373,800 534,700 870,700</td>
</tr>
<tr>
<td>5</td>
<td>380</td>
<td>not broken with 23,160,000 applications of strain.</td>
</tr>
<tr>
<td>6 7 8 9</td>
<td>280 250 240 220</td>
<td>187,500 1,007,550 650,700 19,100,000 applications of strain.</td>
</tr>
<tr>
<td>B 9</td>
<td>220</td>
<td>not broken with 19,100,000 applications of strain.</td>
</tr>
</tbody>
</table>
The Two Stages of Fatigue

Crack size, mm

Crack initiation life
Crack propagation life

Cycles

$\alpha_0$ $a_c$
Primary input to fatigue design

- Strain (elastic, plastic and total) versus fatigue life SAE 4340 steel
Some real load sequences:
TWIST – Transport Wing
FALSTAFF – Fighter Aircraft wing
HELIX – Hinged Rotor load sequence (Helicopter)
TURBISTAN – Typical Mission Profile
Engine Rotary
WISPER – load sequence

The integrity of contemporary wind energy systems depends on the reliability of these data.....
Linear Damage Accumulation – an attempt to account for complex load sequences

Most service load conditions are a mix of different amplitudes

\[ \sum \frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} = D \]

0.5 < D < 10, Miner/Palmgren [1930-45]
Local mean stress can be sequence sensitive!

Inconvenient fact: Sequence sensitivity prevails (but cannot be modeled) even under totally elastic stress-strain response!

\[
\frac{1}{2N_f} = (\frac{\sigma_a}{\sigma_f})^{-1/b} \cdot (1 - \frac{\sigma_m}{\sigma_f})^{1/b} \quad \text{(HCF)}
\]

\[
\frac{1}{2N_f} = (\frac{\varepsilon_{p,a}}{\varepsilon_f})^{-1/c} \cdot (1 - \frac{\sigma_m}{\sigma_f})^{1/b} \quad \text{(LCF)}
\]
State-of-the-art industrial fatigue analysis

- Convert applied elastic load history to notch root inelastic stress-strain
- Compute damage after accounting for sequence-sensitive shift in local mean stress
Modeling Fatigue
Crack Growth
The Stress Intensity Factor

- $K$ describes crack-tip stress field as a function of applied stress, crack size and crack geometry.

\[
\sigma_x = \frac{S \sqrt{\pi a}}{\sqrt{2 \pi r}} \cos \frac{\theta}{2} \left( 1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) - S
\]

\[
\sigma_y = \frac{S \sqrt{\pi a}}{\sqrt{2 \pi r}} \cos \frac{\theta}{2} \left( 1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)
\]

\[
\tau_{xy} = \frac{S \sqrt{\pi a}}{\sqrt{2 \pi r}} \cos \frac{\theta}{2} \sin \frac{\theta}{2} \cos \frac{3\theta}{2}
\]

\[
\sigma_{i,j} = \frac{K}{\sqrt{2 \pi r}} f_{i,j}(\theta)
\]

\[
K = \beta S \sqrt{\pi a}
\]
Validity of the $\Delta K$ Similarity Criterion

- Irrespective of crack-geometry and applied stress level, crack growth rate is uniquely related to applied stress intensity range, $\Delta K$
Fatigue is extremely sensitive to load history

- The same load sequence re-arranged in different ways can cause dramatically different fatigue response.
- Several mechanisms including closure, residual stress, crack-tip blunting, crack front incompatibility, etc., are responsible well, most of the time.....
Effect of overload ratio

Increasing overload ratio causes dramatic increase in delay

K.N. Raju [1979]
Closure explanation for overload effect

- Crack closure requires wake development. Hence delayed retardation COD

KN Raju, 1979
FALSTAFF – Crack growth analysis

Initial crack size $a = a_{in}$

Next rising load excursion, $P_{min}, P_{max}$

Convert to, $\Delta K = K_{min}, K_{max}$

Correct for, crack closure, $\Delta K_{eff}$

Estimate crack extension, $\Delta a = C \cdot \Delta K_{eff}$

Update crack size, $a = a + \Delta a$

(Or, correct for non-linear effects)

Failure?

No
Summary of new inputs to modeling
Significance of hydrostatic stress under plane strain constraint

Even under uniaxial loading, interfacial cracking can commence all around an embedded particulate
Secondary particulate separation through multiple interfacial cracks (Al-alloy)
Embedded crack growth. Vacuum 2014
T6511
Fatigue void formation and embedded crack growth will be relatively insensitive to residual stress. They occur in vacuum. Then, load sequence sensitivity of notch fatigue is exclusively a crack growth – related phenomenon!
Notch root semi-elliptical crack. Initiation at
Long crack, 2014-T6511

No closure.
No sequence effect

Paris regime, $\sim 4 \times 10^{-5}$ mm/cycle. Bands from 2000 cycles between markers
Surface Diffusion of Active Species

Hydrogen locked between substrate atoms embrittles the surface through slip inhibition. Schematic from Bowles PhD thesis.

Tensile hydrostatic stress accelerates diffusion kinetics including interstitial movement.

Gsell et al.

Schematic of argon bubble inserted beneath ruthenium monolayers [23]. The consequent dislocation-free strain redistribution causes differential adsorption of active species, demonstrating sensitivity of surface chemistry to stress distribution. Figure reproduced with kind permission of authors.
How mean stress may *in fact* affect fatigue

Theory of Brittle Micro Fracture (BMF)

Nascent hydrogen released by crack tip surface chemistry is “sucked in” by rising hydrostatic stress

\[
2\text{Al} + 3\text{H}_2\text{O} \rightarrow \text{Al}_2\text{O}_3 + 6\text{H}
\]

\[
\text{Al} + 3\text{H}_2\text{O} \rightarrow \text{Al(OH)}_3 + 3\text{H}
\]

Hydrogen atoms locked within surface layer inhibit slip forcing it to fracture

Hydrostatic stress will be sensitive to history-sensitive hysteretic stress-strain response

[Sunder, 2003]
Significance of hydrostatic nature of stress (1)

Unstressed

Uniaxial (plane stress) stretch

Hydrostatic stretch
Significance of hydrostatic nature of stress (2)

“Unyielding” attitude also embrittles as opposed to toughness associated with yield
Tensile hydrostatic stress (1) enhances diffusion kinetics and (2) reduces ductility. 1 & 2 accelerate Brittle Micro Fracture (BMF)

Nascent hydrogen released by crack tip surface chemistry is “sucked in” by rising hydrostatic stress

\[
2\text{Al} + 3\text{H}_2\text{O} \rightarrow \text{Al}_2\text{O}_3 + 6\text{H} \\
\text{Al} + 3\text{H}_2\text{O} \rightarrow \text{Al(OH)}_3 + 3\text{H}
\]

Hydrogen atoms locked within surface layer inhibit slip forcing it to fracture

Hydrostatic stress will be sensitive to history-sensitive hysteretic stress-strain response

[Sunder, 2003]
da/dN in 2124 Al-alloy under periodic overloads

Absurd fact:

Material resistance is not a constant!!

Effect of periodic overloads dominant in near-threshold region. Diminishes to vanishing proportions above 10-5 mm/cycle

Convenient fact: Surface and interstitial diffusion effects restricted to low growth rates
Hysteretic sequence effect increases with decreasing $\Delta K_{eff}$.

$\Delta K_{th}$ is a history-sensitive variable – not a material constant. It becomes a constant in high-vacuum, where surface diffusion ceases.

Higher $K$, reduced effect

Lower $K$, dramatic effect
Model Schematic

Goal: digitally simulate crack-tip cyclic stress-strain field along with response of stretched wake.....
Model Structure

Wedging of fractured elements causes closure

Instantaneous stress at this point determines $\Delta K_{th}$

$K$ determines driving force, stress determines instantaneous $\Delta K_{th}$ due to local diffusion kinetics
Crack growth analysis – correcting for change in material resistance

1. Initial crack size
   \[ a = a_{in} \]

2. Next rising load excursion, \( P_{min}, P_{max} \)

3. Convert to,
   \[ \Delta K = K_{min}, K_{max} \]

4. Correct for crack closure,
   \[ \Delta K_{eff} \]

5. Update crack-tip local stress-strain,
   \[ \sigma_{min}, \sigma_{max} \]

6. Correct \( \Delta K_{th} \) for local mean (residual) stress

7. Estimate crack extension,
   \[ \Delta a = C. [\Delta k_{eff} - \Delta K_{th}]^m \]

8. Update crack size,
   \[ a = a + \Delta a \]

9. Failure?
   - No
FALSTAFF Simulation

Crack Growth Simulator

Y-σ
G-K\textsubscript{th}
G-S\textsubscript{op}
R-K
Y-K\textsubscript{eff}

Modulus, MPa: 73000
K', MPa: 1280.0
Spec. Geom.: C(T)
Spec. width, mm: 20.0
Crack Size, mm: 4.0006
Cycles: 1000
Reversals per pass: 2000
TWIST Simulation

Crack Growth Simulator

Y-σ
G-K_{th}
G-S_{op}
R-K
Y-K_{eff}

Modulus, MPa | 73000
K', MPa | 1280.0
Spec. Geom. | C(T)
Spec. width, mm | 20.0
Crack Size, mm | 4.0007
Reversals per pass | 2000

Cycles | 1000
50μm

a = 1.5 mm

Steps:
1, 2, 4, 5 - 5,000 cycles
3 - 2,500 cycles
Simulated crack-tip stress-strain response

Closure inhibits hysteretic mean-stress variation in 1,5
3-Step Hysteresis

- $Y - \sigma$
- $G - K_{th}$
- $G - S_{op}$
- $R - K$
- $Y - K_{eff}$
Goal: microscopic consistency in modeling...
Impact

1. BMF Theory explains the science behind early fatigue – “It’s also about chemistry under near-elastic conditions, not merely cyclic slip!”

2. A common new handle to many problems: small cracks, thresholds, constraint effects, GCF, MEMS, bio-medical applications, etc
Fatigue void formation and embedded crack growth will be relatively insensitive to history effects. They occur in vacuum. Then, load sequence sensitivity of notch fatigue is exclusively a crack growth – related phenomenon! e.g. Hysteretic effect
Hind sight - 20:20

A re-look at history of fatigue research
Fatigue as a process of embrittlement – a highlight of the first 75 years

Understanding over first 75 years of Fatigue Research

Service usage leads to progressive loss of “fibrous quality” of metals. As a consequence, the material can fail “along crystallographic interfaces because their adhesive strength is far less than that of atomic bonds within a crystal”

Material transformation (embrittlement) as root cause....
Highlight of last 75 years of research....

Fatigue as a consequence of cyclic plastic deformation....
Major positives in contemporary fatigue design:

2. Excellent reproduction of actual crack-tip elasto-plastic response (FEM-based fracture mechanics analysis)
3. Advances in characterizing service usage. Reasonably accurate simulation of nominal as well as local stress and strain response to actual usage conditions.

1. Safe designs using fatigue limit based analysis – for very high life and very low life.
2. Reasonable estimates of life between inspections.
3. Excellent residual strength analysis.

Inconvenient fact: Advances restricted to characterizing inputs & onset of fracture. Little progress in predicting early fatigue and low growth rates
1. Early suggestions (1850) of brittle (crystalline) failure dismissed (1950) in deference to the elegance of dislocation dynamics, slip as seen through electron microscopy and low-cycle fatigue testing.

2. Constraint effect including that of diffusion enhanced by hydrostatic stress largely ignored. **Inconvenient fact:** Dearth of data on material response under constraint.


4. Painted into a corner by assumptions required for ease of modeling:
   - Elastic, ideally plastic crack tip response – $R = -1$, no near tip history effect, no hysteresis
   - Fracture Mechanics parameter, $K$, dominates – no recourse to crack tip cyclic stress-strain analysis
   - Closure as single mechanism controlling sequence effects (amenable to modeling) – ignoring all other load interaction models. **Inconvenient fact:** exaggerated closure levels to explain real data.
Summary of technologies used in this work

1. Specially designed programmed load sequences to facilitate microscopic marking of fracture surface
2. Accent on characterizing low growth rates under the influence of load sequence effects (they determine life)
3. Reference tests in vacuum
5. First attempt to simulate more than one operative load interaction mechanism in fatigue
Dispensable baggage

“Crack-tip response may be modeled as elastic – ideally plastic, i.e., crack-tip R = -1”

“K-threshold is a material constant determined by ASTM E-647”

“$da/dN$ data will collpase into a single band once ‘suitably’ corrected for driving force (closure, J, $K_{PR}$, etc)”

“Crack initiation and propagation demand independent and separate approaches”
Future Effort

Materials science: Surface diffusion kinetics and surface chemistry including oxidation as a function of hydrostatic stress in engineering materials

Mechanics: Modeling near-tip hydrostatic stress as a function of applied load history

Interdisciplinary research to model instantaneous local material resistance

Test technology to impose controlled environment in fatigue tests under complex loading

Till then: Experimental determination of $\Delta K_{th}$ as a function of simulated near-tip hydrostatic stress